

A STUDY ON PRODUCT OF COMPLETE BIPARTITE GRAPHS OF THE FORM  
 $K_{n,n}; n \in \mathbb{Z}^+$

M.D.M.C.P. Weerathna<sup>\*</sup>, D.M.T.B. Dissanayake, A.M.U.J.B. Abeysinghe,  
M.H.M.I. Kumari and A.A.I. Perera

*Department of Mathematics, Faculty of Science, University of Peradeniya, Peradeniya, Sri Lanka*  
*\*chinthakaweerathna@gmail.com*

The study of graphs with number theory is a flourishing research area in graph theory. In graph theory, new graphs can be generated from existing graphs using a well-defined set of rules. One such way of defining a combination graph is via a binary operation. Graphs generated in such manner can have applications in Mathematics, Communication theory and Network analysis. In current literature, it can be found methods to translate arithmetic operations for the natural number system into complete graphs. This study focuses on the multiplication of complete bipartite graphs of the form  $K_{1,1}$  and  $K_{2,2}$ , which is defined using a bijection between complete bipartite graphs and the set of natural numbers. A simple graph is called a complete bipartite graph, if its vertices can be partitioned into two disjoint subsets in such a way that no edge joins two vertices in the same set, and if its every vertex in one partite set is adjacent to all the vertices in the other set. If the two partite sets have cardinalities  $s$  and  $t$ , then the complete bipartite graph is denoted by  $K_{s,t}$  where  $s, t \geq 1$ . In this study, it was proved that by incorporating multiplication operation of the natural number system into the complete bipartite graph of the form  $K_{n,n}$ , where  $n \geq 1$ , a higher-order degree of complete bipartite graph can be constructed. The multiplication operation was defined for the complete bipartite graphs  $K_{n,n}$ , where  $n$  is the number of vertices in one partite set. The results  $(K_{1,1})^n = K_{2^{n-1}, 2^{n-1}}$  and  $(K_{2,2})^n = K_{2^{2n-1}, 2^{2n-1}}$  for  $n \in \mathbb{Z}^+$ , were established for the product of the complete bipartite graphs  $K_{1,1}$  and  $K_{2,2}$  respectively. Furthermore, the result  $(K_{1,1})^{2k} = (K_{2,2})^k$  was obtained.

**Keywords:** Complete bipartite graphs, Generating graphs, Natural numbers