

## GENERALIZED HADAMARD MATRICES AND 2-FACTORIZATION OF COMPLETE GRAPHS

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Graph factorization plays a major role in graph theory and it shares common ideas in important problems such as edge coloring and Hamiltonian cycles. A factor  $F$  of a graph  $G$  is a spanning subgraph of  $G$  which is not totally disconnected. An  $n$ -factor is an  $n$ -regular spanning subgraph of  $G$  and  $G$  is  $n$ -factorable if there are edge-disjoint  $n$ -factors  $F_1, F_2, \dots, F_k$  such that  $G = F_1 \cup F_2 \cup \dots \cup F_k$ . We shall refer  $\{F_1, F_2, \dots, F_k\}$  as an  $n$ -factorization of a graph  $G$ . In this research we consider 2-factorization of complete graph. A graph with  $n$  vertices is called a complete graph if every pair of distinct vertices is joined by an edge and it is denoted by  $K_n$ . We look into the possibility of factorizing  $K_n$  with added limitations coming in relation to the rows of generalized Hadamard matrix over a cyclic group. Over a cyclic group  $C_p$  of prime order  $p$ , a square matrix  $H(p, v)$  of order  $v$  all of whose elements are the  $p^{\text{th}}$  root of unity is called a generalized Hadamard matrix if  $HH^* = vI_v$ , where  $H^*$  is the conjugate transpose of matrix  $H$  and  $I_v$  is the identity matrix of order  $v$ . In the present work, generalized Hadamard matrices  $GH(3, 3^m)$  over a cyclic group  $C_3$  have been considered. We prove that the factorization is possible for  $K_{3^m}$  in the case of the limitation 1, namely, if an edge  $\{i, j\}$  belongs to the factor  $F_k$ , then  $i^{\text{th}}$  and  $j^{\text{th}}$  entries of the corresponding generalized Hadamard matrix should be different in the  $k^{\text{th}}$  row. In Particular,  $\frac{(n-1)}{2}$  number of rows in the generalized Hadamard matrices is used to form 2-factorization of complete graphs. We discuss some illustrative examples that might be used for studying the factorization of complete graphs.

**Keywords:** Factor, Factorization, Generalized Hadamard matrices, Kronecker product