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BEHAVIOUR OF PURE ALGEBRAIC ISOPAIRS AT NON-REGULAR POINTS

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A polynomial in z and w is called inner toral if its zero set is a subset of $\mathbb{D}^2 \cup \mathbb{T}^2 \cup \mathbb{E}^2$, where $\mathbb{D}$ is the open unit disk, $\mathbb{T}$ is the unit circle, and $\mathbb{E}$ is the exterior of the closed unit disk. An inner toral polynomial is called a minimal inner toral polynomial if it divides any polynomial with the same zero set as itself. A zero of a polynomial $p(z, w)$ is called a regular point for p, if the gradient of p at that point is non-zero. An isometry defined on a Hilbert space is called a pure isometry if it behaves like a shift operator. The bimultiplicity of a pair (S, T) of pure isometries S and T is given by $(\dim(\ker(S^*), \dim(\ker(T^*))$, where * denotes the adjoint of an operator. Given a minimal inner toral polynomial $\mathfrak{p}$, a pair of pure isometries (S, T) satisfying the algebraic relationship $\mathfrak{p}(S, T) = 0$ is called a pure $\mathfrak{p}$-isopair. In 2018, it was proved that, for a pure $\mathfrak{p}$-isopair (S, T) with finite bimultiplicity, $\dim[\ker(S - \lambda I)^* \cap$ $\ker T - \mu I * = 1$ whenever $\lambda, \mu \in \mathbb{D}2$ is a regular point for p. In this paper, we show that the converse of this result does not hold. The pair (M_z, M_w), is a pure $\mathfrak{p}$-isopair with finite bimultiplicity $(2, 2)$, where $\mathfrak{p}$ is taken to be as the minimal inner toral polynomial $z^2 - w^2$, and M_z and M_w are multiplication by z and w, respectively. We show that the dimension of $[\ker(M_z - \lambda I)^* \cap \ker(M_w - \mu I)^*]$ is 1 at the point $(\lambda, \mu) = (0,0)$, which is a non-regular point for $\mathfrak{p}$.

Keywords: Algebraic isopairs, Inner toral polynomial, Isometries